3201. Prove this by construction, explicitly finding the centre of rotation. Expressing the quadratics in completed square form (invent new constants for this purpose) is the easiest way to do this.

3202. (a) Consider the $(n-k)$ lowest links as a single object. Use NII.
(b) Consider the limit as the size of the links in a chain becomes very small. You can set $a = 0$ or not as you wish.

3203. For a combinatorial approach, the possibility space has ${}^8\mathrm{C}_4$ total outcomes.

——— Alternative Method ———

For a conditioning approach, choose a vertex wlog. Then consider the number of successful locations for the second vertex.

3204. Rearrange to the form $y = \mathrm{f}(k)x$, and show that the range of $\mathrm{f}(k)$ is $\mathbb{R}$. Also show that $x = 0$ is one of the lines.

3205. This is a cubic in 2^x. Use the substitution $z = 2^x$ and then the factor theorem. You might want to use a polynomial solver to help you, even if it won't show anything.

3206. Differentiate with respect to y, and thus find the equation of the tangent line. Solve simultaneously for intersections.

3207. (a) Use a double-angle formula, and then one of the Pythagorean identities.
(b) Use the identity in part (a), and then solve using the factor theorem.

3208. Write the solution as a pair of equations for z and y in terms of x, with constants of proportionality k_1 and k_2. Differentiate each with respect to x.

3209. Divide $(x^4 + 3x^2)$ by $(x + 1)$. A "proper algebraic fraction" means that the degree of the numerator should be lower than that of the denominator.

3210. Solve simultaneously for intersections, and show that the resulting equation has a triple root (point of tangency).

3211. Consider the sign of the first derivative.

3212. (a) Use a calculator.
(b) Find the probability that neither sablefish is over 70 cm long, and subtract this from 1.

3213. Neither of these holds.

3214. Expand with compound angles, and then use small angles. The assumptions concern the unit of angle and which terms are to be neglected.

3215. Translate the entire problem by vector $b\mathbf{i}$ before you begin. Then find the equation of the tangent at $x = 2$. Solve this equation simultaneously with that of the curve.

3216. Use the substitution $u = 1 + \sqrt{x}$.

3217. Allocate role by role, multiplying the numbers of ways in which each set of roles can be filled.

3218. Expand the LHS with a compound-angle formula and simplify to $(1+\tan\theta)/(1-\tan\theta)$. Expand the RHS with double-angle formulae, before putting the fractions over a common denominator. Then factorise top and bottom.

3219. (a) Use the cumulative normal distribution on a calculator.
(b) Use $\mu = np$ for a binomial distribution.
(c) Use the inverse normal facility on a calculator, with $p = 0.99$ and $\sigma = 0.02$.
(d) Do likewise, but now with $\sigma = 1$.

3220. Compare the degree and number of known roots of the polynomial equation $\mathrm{f}(x) = \mathrm{g}(x)$.

3221. Express the numbers as $a = 0.A$[other digits] and $b = 0.B$[other digits]. Show that $\frac{B}{10}$ satisfies the conditions of the proof.

3222. Consider this sum with reference to the unit circle or a sine graph: no calculation is required.

3223. Assume the same μ at both supports. Draw a force diagram for the rod, and consider the magnitude of the reaction (and thus friction) at each support.

3224. (a) Consider the set of inputs to the question.
(b) Consider the type of answer that emerges.
(c) Consider whether any elements of your set from (b) are unattainable.

3225. Consider the formula for S_{xx}.

3226. By considering the inputs x and $-x$, show that the graph has the y axis as a line of symmetry.

3227. (a) Count outcomes.
(b) The fact that the central region is blue gives no information as to the probability of there being exactly two colours.

3228. (a) Differentiate.

(b) Find a rational root $x = {}^{b}/_{a}$, and so deduce a factor $(ax - b)$.

(c) Take out the factor you found in (b), and so find the remaining factors, or use the method of (b) to find the remaining factors.

3229. This is best done using circle geometry (as opposed to coordinate geometry). Note that the two circles have centres on the x axis, so the tangents must meet on the x axis.

3230. You can prove this algebraically, using

$$a, a+k, a+2k, a+3k \quad \text{and} \quad \mathrm{f}(x) = px + q.$$

But it's easier visually, considering the fact that the points $x = a, b, c, d$ are spaced equally along the x axis.

3231. In each case, find the second derivative and then factorise it. Hence, work out whether it becomes zero and changes sign anywhere.

3232. This is a counting exercise: find a way (easiest on a set of axes) to list the possibilities. Note that $\mathbb{Z}^2$ is just a notation for the lattice of integer grid points in the (x, y) plane.

3233. (a) Express x and y in terms of t, and eliminate t.

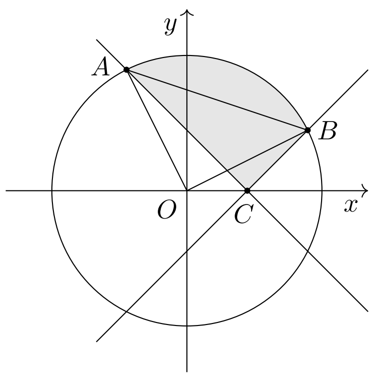
(b) Use your answer from (a).

(c) No major calculation is required here. Use the symmetry of a parabola.

3234. Set up an expression for the y distance between the curves. To show that it has no roots, consider the amplitude of the combined trig functions (i.e. in harmonic form).

3235. The sum of n standardised, independent normal variables has distribution $N(0, n)$.

3236. The region R in question, subdivided by chord AB into a segment and a triangle, is shaded below:



3237. The angle of inclination of L is θ, and it passes through the point (a, b). Draw a clear sketch. The reflection(s) will also pass through (a, b).

3238. (a) Consider that, near the y axis, $|x|$ is small.

(b) Differentiate implicitly to find a formula for $\frac{dy}{dx}$. Set the numerator/denominator of this to zero to look for horizontal/vertical tangents.

(c) Consider the behaviour as $x \to \pm\infty$. Combine this with the results of (a) and (b).

3239. Each factor is the same quadratic, either in x or x^3. Consider the discriminant.

3240. This can be calculated using the infinite sum of a GP. However, it is much easier to condition on "the throw on which the game ends", which must come eventually.

3241. Expand the compound angles in the first equation, and simplify to get two equations both involving $\sin\theta$. Eliminate $\sin\theta$, and solve for ϕ.

3242. One is true, two are false.

3243. Rearrange to make y the subject, then consider symmetry, axis intercepts and the behaviour as $x \to \pm\infty$.

3244. Find $|OA|$, $|OC|$, $|AB|$ and $|BC|$ in terms of θ, using similar triangles. Then apply Pythagoras to triangles OAB, OCB and OAC. The first two give standard Pythagorean identities, the third a less familiar one.

3245. Find and simplify an expression for the squared distance between the asteroids. Use calculus or complete the square to minimise this.

3246. (a) Use the quotient rule, and then set the first derivative to zero.

(b) Show that the second derivative is zero and changes sign at the origin.

(c) Use parts (a) and (b).

3247. Use the substitution $x = a^2 - 1$.

3248. (a) Draw a force diagram, including acceleration a down the slope.

(b) Calculate a, and use it in a *suvat*.

(c) Take the answers from (b), and set them as the initial conditions for projectile motion.

3249. The DE is satisfied for any graph whose gradient is either 1 or 2.

3250. Write the trig functions in harmonic form. In other words, find R and α such that

$$\sin t + \left(\sqrt{2} - 1\right)\cos t \equiv R\sin(t + \alpha).$$

Then solve for t.

3251. Determine the range of $\sec x$ first, before setting this as the domain of $x \longmapsto e^x$.

3252.
(a) Set the first derivative to zero.
(b) Consider your answers to (a), together with the domain of the graph, the behaviour for large x, and any axis intercepts.

3253. Throughout, a clear diagram with help.
(a) Consider the translation as a weighted average of the translations from A to B and A to C.
(b) Write all of the vectors in terms of $\mathbf{a}, \mathbf{b}, \mathbf{c}$.
(c) Use both (a) and (b).

3254. Put the fractions over a common denominator, and then use Pythagorean trig identities.

3255.
(a) Set $(a\sqrt{3} + b)^2 = 8 - 4\sqrt{3}$, multiply out and equate coefficients.
(b) Reciprocate and find a simplified expression for $\tan\phi$, then use the second Pythagorean identity and part (a). The fact that ϕ is acute allows you to choose between the square roots.

3256. In one sense, the result is obvious, thinking about the equation of a straight line

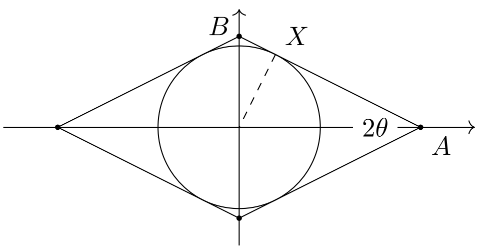
$$m = \frac{y - y_1}{x - x_1}.$$

But we can't assume that $\frac{dy}{dx}$ is a constant m, so it isn't easy to make such an argument rigorous.

More reliable is to solve the DE by separating the variables. Consider the point $(1, 1)$ once you have the general solution.

3257. Consider a situation where the individual samples have very different means, compared to the sizes of the samples.

3258. The rhombus is as follows:



Find the coordinates of X in terms of θ. Using similar triangles, show that the perimeter is given by

$$P = 4(\cot\theta + \tan\theta).$$

Simplify this using trig identities.

3259.
(a) use a double-angle formula to write x in terms of $\cos 2y$.
(b) This graph is a subset of that in (a).

3260. Take logs with base k of both sides of the given prime factorisation statement. Simplify using log rules.

3261. Draw a clear sketch of the first two forces, which includes their lines of action. For an object to be in equilibrium under the action of three forces, the three lines of action must be concurrent.

3262. Take logs base 2 of the equation given, and then use log rules to rearrange.

3263. Since a_p can take any value in the range (over $\mathbb{N}$) of $20n - n^2$, and likewise b_q, the question suggests that there must only be a single value which lies in the range of both quadratics. Work this out by completing the square or differentiating. A sketch may also help, as ever!

3264. Consider the symmetry of $y = \text{f}(x)$. Since f is odd (no even powers), the graph has rotational symmetry around the origin. Hence, since it is a polynomial, it passes through the origin.

3265. Since the triangle is right-angled, the area is $\frac{1}{2}ab$. For this to be an integer, you need to show that at least one of a and b has a factor of 2. Prove this by contradiction.

3266.
(a) Consider the second derivative as the gradient of the graph shown.
(b) Show that the cubic has a stationary point of inflection.

3267. Solve the equations simultaneously for $\mathbf{a}$ and $\mathbf{b}$, in terms of $\mathbf{i}$ and $\mathbf{j}$. You can then write these as column vectors.

3268.
(a) There is one successful outcome.
(b) There are two successful outcomes, depending on the random number chosen.

3269. Resolve along the angle bisector, with angle $\theta/2$. Then use $\cos^2 x \equiv \frac{1}{2}(\cos 2x + 1)$, with $x = \frac{1}{2}\theta$.

3270. The equation is a quadratic in xy. Solve it.

3271. (a) Find a counterexample.
(b) Find explicit expressions for the RHSs, in the cases that x is positive or negative.

3272. Assume, for a contradiction, that there exists a smallest positive irrational number k. Then find a smaller positive irrational number.

3273. Write $\frac{u}{v}$ as uv^{-1}, then differentiate by the product and chain rules.

3274. Show first that the amount owed after 1 month is

$$T_1 = P(1+r) - c.$$

Then apply the interest rate to the outstanding amount, subtract another c, and simplify.

3275. Solve the inequality to find the successful values of Z. Then use a normal distribution calculator to evaluate the probability.

3276. Use the first Pythagorean identity.

3277. Use the fact that the second square occupies the interval $y \in [0, 1]$.

3278. (a) Draw a tree diagram, conditioning on having had the disease D or not D'. Let $p = \mathbb{P}(D)$. Set up and solve an equation in p.
(b) 30% is a value from a *sample*.

3279. Factorise $e^{2x} - 1$ as a difference of two squares.

3280. (a) Use the chain and then product rules.
(b) The greater the curvature (i.e. magnitude of the second derivative), the more the curve will depart from any line used to approximate it.

3281. Consider $y = ax^2 + bx + c$. You need to reflect this in the y axis, and also stretch it, scale factor 2, in the x direction. In combination, this is a stretch, scale factor -2, in the x direction.

3282. This is a quadratic in $(x - 2y)$. Factorise.

3283. Express both probabilities in terms of r and b, and set up a pair of equations. Multiply up so both equations involve $(b+r)(b+r-1)$. Substitute for this, and solve.

3284. This is true. Rewrite the second implication, and it becomes obvious.

3285. (a) Set t to be e.g. $\frac{2}{3}$ for your sketch.
(b) Find the equation of PQ and of the normal to PQ through O. Solve these simultaneously.
(c) Use Pythagoras, taking out and cancelling common factors when you can.
(d) You don't need to do any algebra: consider the symmetry of the original problem.

3286. Consider the value p as generating translations of the curves $y = x^3$ and $x = y^3$.

3287. Find the area A_1 of the central region first. It can be subdivided into a square and four segments. Show that each of its segments subtends 30°. Use this to find the positions of the intersections.

Then find the area A_2 of a region common to two diagonally opposite right-angled sectors.

The shaded area is $2A_2 - 2A_1$.

3288. Use the small-angle approximations $\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \frac{1}{2}\theta^2$.

3289. The final calculation has been given as if this were a *two*-tailed test.

3290. Either use the discriminant or circle geometry.

3291. (a) Consider the number of steps taken to reach the nth term.
(b) Find rS_n. When you calculate $S_n - rS_n$, all but two of the terms should cancel.

3292. Use the substitution $z = x^2 - 4$.

3293. (a) Consider limiting equilibrium (i.e. friction at F_{max}) for the upper block.
(b) Consider limiting equilibrium (i.e. friction at F_{max}) for the lower block.
(c) Combine your answers from (a) and (b).

3294. Let $u = \cos x$. Find $\frac{du}{dx}$ and therefore $\frac{dx}{du}$ in terms of x. Then find $\frac{d}{du}(\sin x)$ by the chain rule (that is to say, implicit differentiation), and substitute the relevant derivative in.

3295. In each case, take out a factor of $(x - 1)$ from the both top and bottom. Cancel this factor before taking the limit.

3296. (a) Use a calculator.
(b) Find the distribution $X_1 + X_2$ using standard results regarding the normal distribution, then use a calculator.

3297. Show that the gradients are reciprocals, i.e. that the lines are reflections in $y = x$.

3298. You need to show that the line doesn't intersect the boundary parabola, and that, therefore, it lies below it for all x.

3299. (a) Use Pythagoras, dividing the triangle in two.
 (b) Eliminate b and rearrange.
 (c) Turn the result from (b) into an iteration, and run it with e.g. $a_0 = 10$.

3300. Simplify with log rules. To simplify the second term, write $\log_4$ as $\log_2$.

——— End of 33rd Hundred ———